Statistics of FORTE Noise between 29 and 47 MHz

T. J. Fitzgerald,
Los Alamos National Laboratory Los Alamos, New Mexico

Abstract

The FORTE satellite triggered on and recorded many radio-frequency signals from low earth orbit. In addition to the triggering signal FORTE also recorded the background noise environment. One would expect the background noise in its subbands to be Gaussian except at frequencies which are dominated by carriers. I have examined the distribution of noise power in 134 records that were triggered by LAPP shots. I filtered the data into 8 subbands of 1 MHz width and excised the portions containing the LAPP signal. I then examined the distribution of the power in the remaining data and compared it to that expected for a gaussian noise (Rayleigh). The recorded distribution is close to Rayleigh but a Nakagami-Rice distribution is a closer fit. The observed distribution would predict a lower average alarm rate than that expected for a Rayleigh distribution.

1 Introduction

For a trigger system such as that employed by the FORTE radio-frequency instrument, the alarm rate depends on the trigger threshold level relative to the distribution of the noise power. One would expect the background noise in the trigger subbands to be Gaussian except at frequencies which are dominated by carriers. That is, the noise at each subband is a sum of many random man-made signals. One might also expect even at frequencies dominated by high-power broadcasters such as the video of VHF television that there would be a sufficient number of sources to again approximate a Gaussian distribution. More troublesome would be bands that contain impulsive signals such as those attribituted to radars.

I decided to test the hypothesis by examining FORTE noise statistics for times when FORTE was over the United States. The FORTE satellite triggered on and recorded many radio-frequency signals from low earth orbit. In addition to the triggering signal FORTE also recorded the background noise environment. The FORTE system typically captures a waveform record lasting 409 μs in a band between 28 and 49 MHz; the capture is initiated by a trigger system which requires coincidence of the signal in 5 out of 8 subbands within the 20 MHz range. The waveform is digitized at a 50 MHz rate and stored in memory for later retrieval.

2 Method

I examined the distribution of noise power in 134 records that were designated as having been triggered by pulses from the Los Alamos Portable Pulser (LAPP). These records were obtained in June, 1998. The subsatellite points lie in the general area of North America although it is likely that during the data collects FORTE saw significantly different noise environments. It is unlikely that there were other broadband, impulsive signals in the data record although some may contain narrowband impulses. I filtered the data into 8 subbands of 1 MHz width and excised the portions containing the LAPP signal.

2.1 FORTE Data Records

Figure 1 shows the time series and spectrogram of a typical data record that was analyzed.

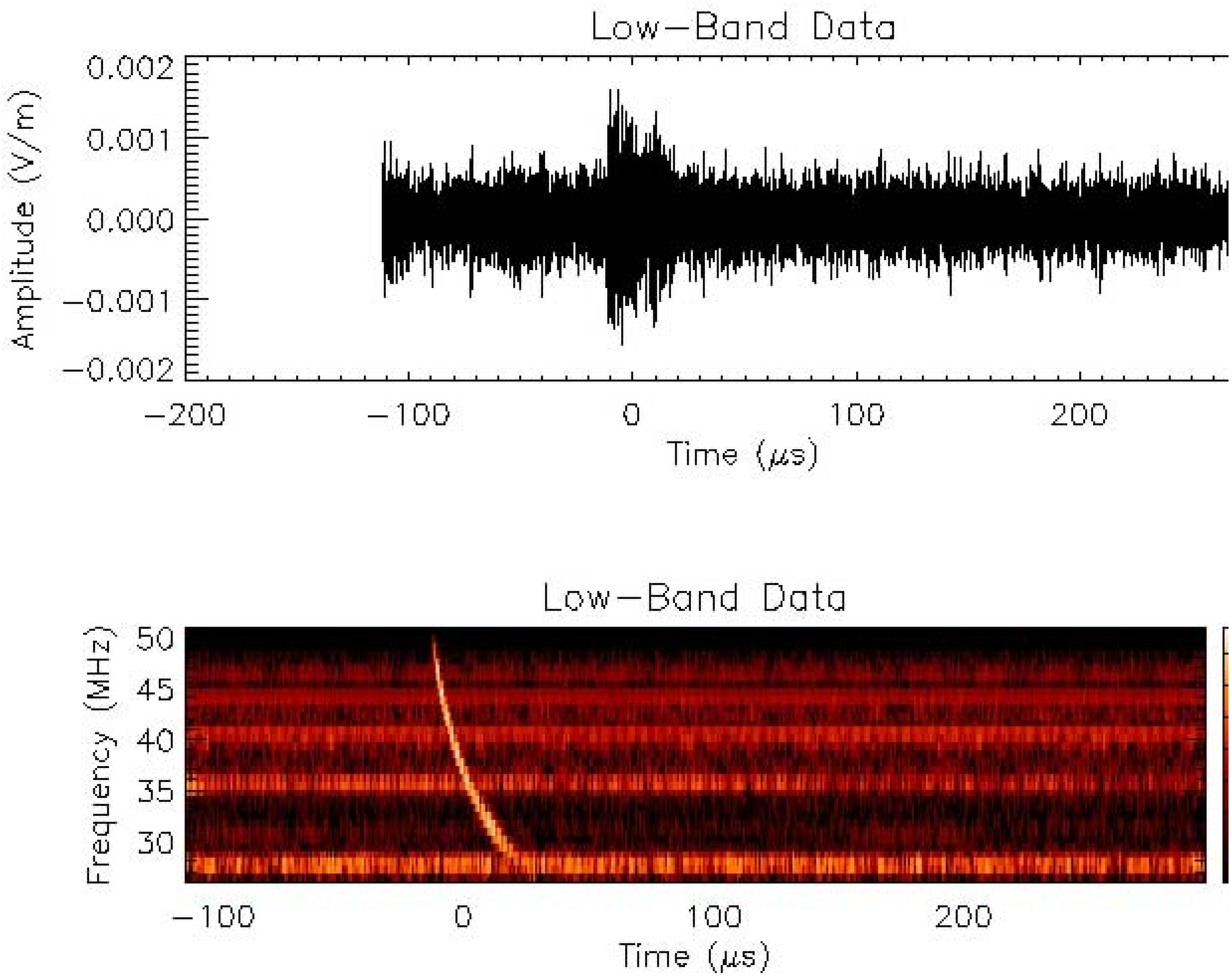

Figure 1: Time series and spectrogram of one of the LAPP shots from June, 1998, that was analyzed.

This is low band data with the usual trigger box settings: 5 out of 8 subbands, 18 dB noise-riding threshold. There was obviously no subband triggers before the LAPP signal. There is some bias therefore in the data set but the chance of noise initiating a trigger at an 18 dB threshold is very small.

2.2 Subband Filtering

I filtered the FORTE data into subbands with Gaussian profiles and a width of 1 MHz at -3 dB. I did this by basebanding the data from the selected frequency and then convolving it with a Gaussian time domain filter. That is, if s is the original time series, I formed two series for a subband of frequency f : $[x',y'] = [s\cos 2\pi ft, s\sin 2\pi ft]$. I then formed a low-pass filtered version by convolution with a Gaussian:

$$[x,y] = [x',y']* \frac{\sqrt{2}\exp(-\pi^2 t^2 B^2/.34657)}{\int \exp(-\pi^2 t^2 B^2/.34657)dt}$$

where B is half the 3 dB bandwidth (.5 MHz). The subbands were spaced 2.5 MHz apart beginning at 29 MHz and ending at

46.5 MHz. In the processing I resampled the data from 50 MHz to 2 MHz. Figure 2 shows an example of the filtered time series in one subband.

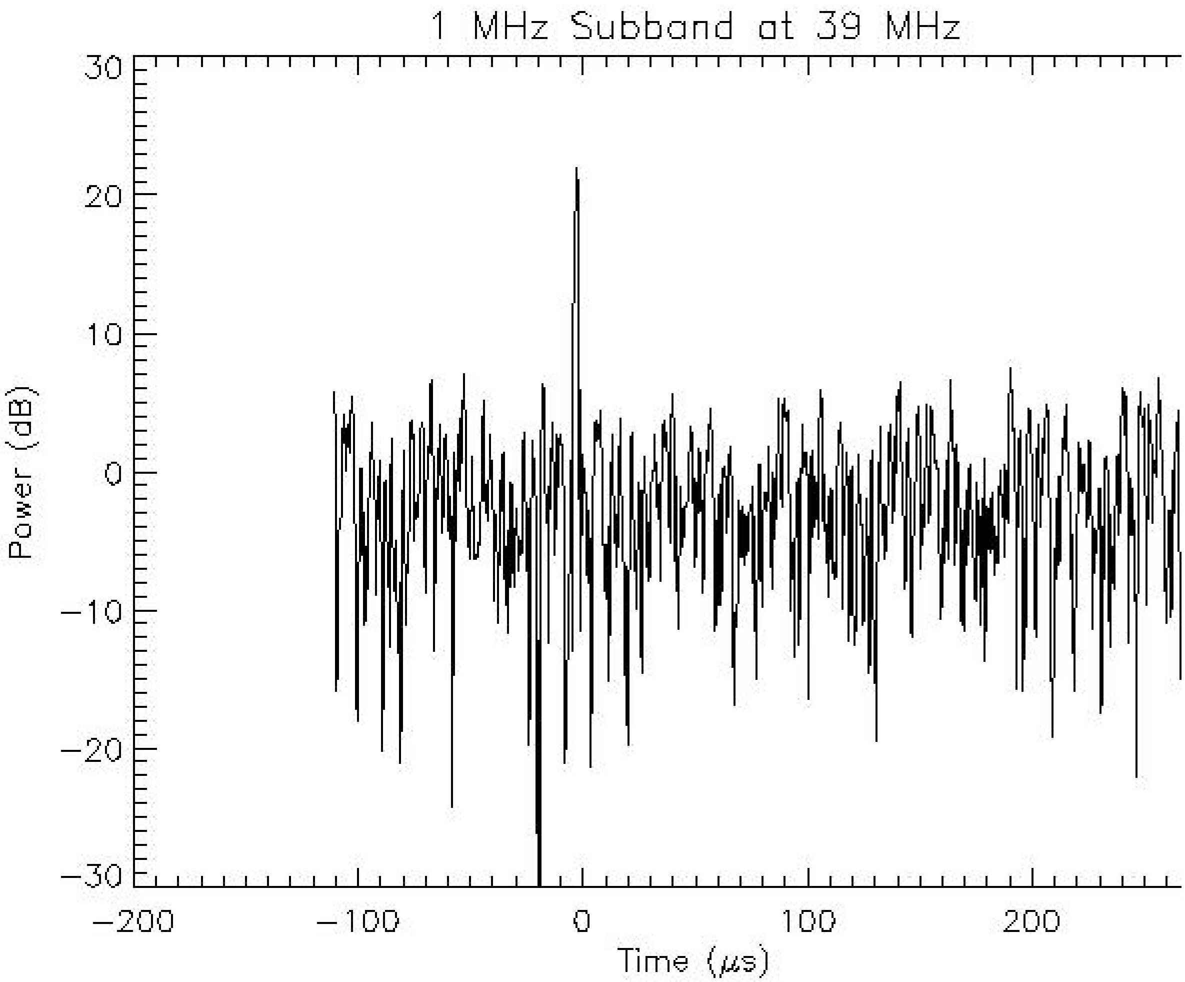

Figure 2: Power versus time in filtered subband at 39 MHz. The filter had a Gaussian profile and a width of 1 MHz at -3 dB. The peak near zero time indicates the power from the LAPP.

2.3 Averaging

I found the trigger time in each subband dataset and excised the period from -50 μs to +100 μs relative to that time. I then found the average power in the remaining data, converted it to dB and subtracted that from the dataset in dB. I then binned the data consisting of 517 points into 1 dB increments from -30 to +19 dB.

3 Results

I combined the calculated distributions for each subband and for all 134 records into one distribution of 747,000 samples.

3.1 Noise Distribution

Figure 3 shows the total distribution versus power.

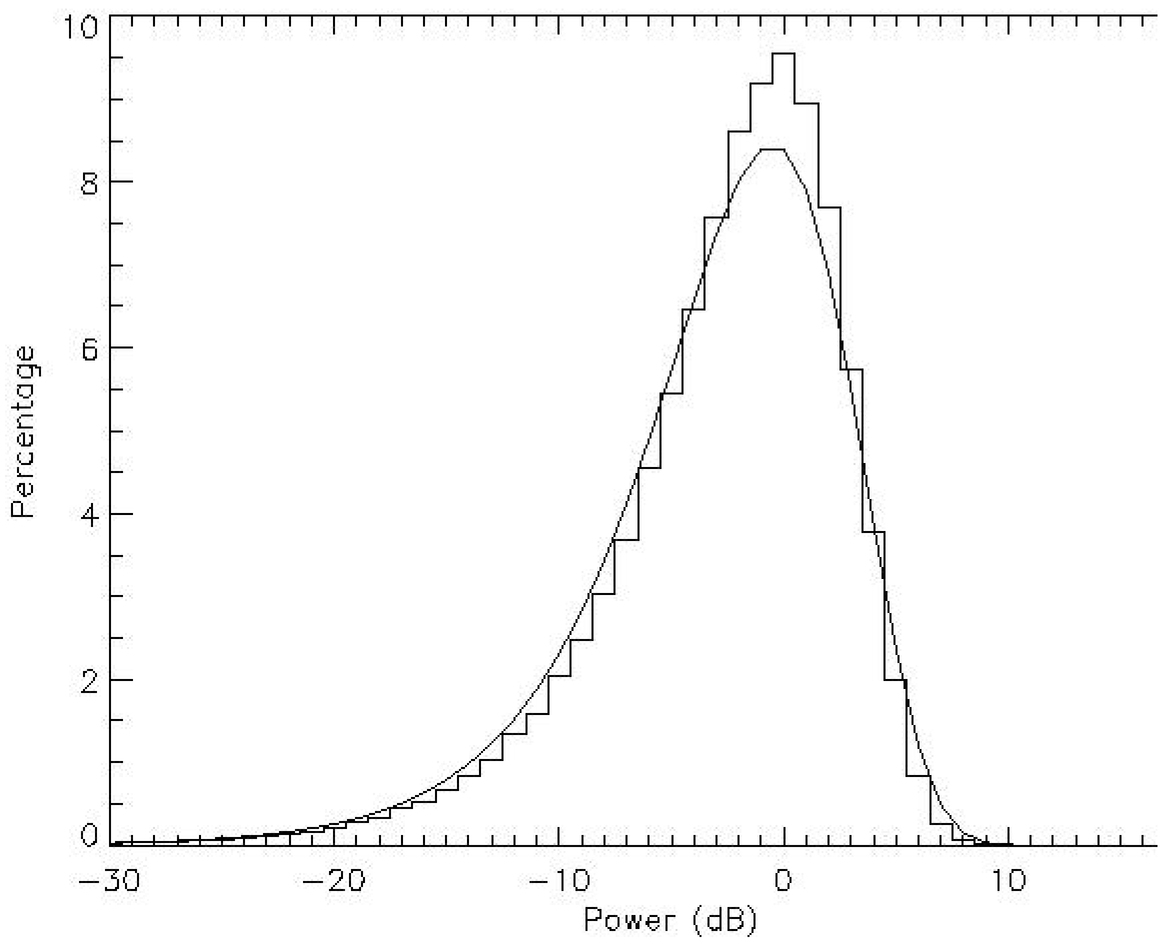

Figure 3: Histogram of the distribution of noise power. The smooth curve is the Rayleigh distribution.

3.2 Rayleigh Distribution

If our data has the properties $< x > = < y > = 0$ and $< x^2 > = < y^2 > = \sigma^2$ then the distribution is given by[1]

$$dP(x,y) = \frac{1}{2\pi\sigma^2} e^{-[(x^2+y^2)/(2\sigma^2)]} dxdy$$

Defining $z = x^2+y^2$ and $\gamma = 2\sigma^2$ one obtains the Rayleigh power distribution

$$dP(z) = \frac{1}{}$$

$$\gamma\, e^{-[(z)/(\gamma)]} dz$$

and the cumulative distribution between $z = 0$ and z is

$$P(z) = 1 - e^{-[(z)/(\gamma)]}$$

The smooth curve in Figure 3 shows the distribution versus power in dB relative to γ.

4 DISCUSSION

The recorded distribution is close to a Rayleigh. However, the χ^2 statistic is 1.13×10^4 which represents a significant departure from Rayleigh. The observed distribution appears to over represent values of power near 0 dB and under represent values greater than 5 dB. It is likely that the FORTE noise is better represented by a random Gaussian process superimposed on narrow band transmissions; such noise has a Nakagami-Rice distribution rather than Rayleigh [2]. That is, the distribution of the signal amplitude, r, is described by

$$dP(r) = \frac{2r}{\gamma} e^{-[(r^2+c^2)/(\gamma)]} I_0\left(\frac{2rc}{\gamma}\right) dr$$

where $C = c^2/2$ is the signal power in the constant component and I_0 is the modified Bessel function. The distribution of the signal power is

$$dP(z) = \frac{1}{\gamma} e^{-[(z+2C)/(\gamma)]} I_0\left(\frac{2\sqrt{2zC}}{\gamma}\right) dz$$

Figure 4 compares the noise distribution, *dP(z)*, of a Nakagami-Rice distribution with $C = .4$ and $\gamma = 1$ to the Rayleigh distribution. The Nakagami-Rice distribution more closely matches the observed distribution in Figure 3.

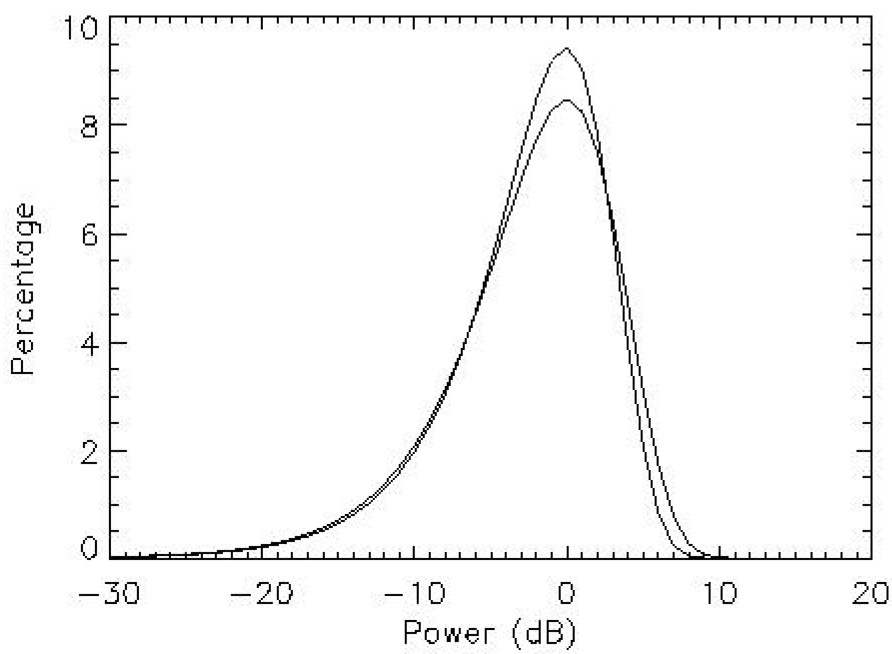

Figure 4: Comparison of a Nakagami-Rice distribution with $C = .4$ and $\gamma = 1$ (thick curve) to the Rayleigh distribution. The

Nakagami-Rice distribution more closely matches the observed distribution in Figure 3. The Nakagami-Rice distribution was shifted by -3 dB so that the power at the peak of both distributions was the same.

4.1 Alarm Rates

The alarm rate produced by noise in a single channel trigger is approximately equal to the bandwidth times the fraction of the time the noise is above the threshold. For a noise-riding threshold that fraction is given by the integral of the distribution above the threshold meashred relative to the average power. From Figure 4 we can see that the observed noise distribution will have a significantly lower alarm rate at 10 dB threshold than that for a Rayleigh distribution. For a 1 MHz channel at 10 dB threshold, a Rayleigh distribution would give an alarm rate of 1150 Hz while the Nakagami-Rice distribution with the observed parameters would give 2 Hz.

References

[1] Siddiqui, M. M., Some problems connected with Rayleigh distributions, *J. Res. NBS*, *66D*, 167-174, 1962.
[2] Beckman, Petr, Rayleigh distribution and its generalizations, *J. Res. NBS*, *68D*, 927-932, 1964.

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